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Modeling and Optimization of Compressive Strength of Hollow Sandcrete Block with Rice Husk Ash Admixture

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Abstract

The paper presents the report of an investigation into the model development and optimization of the compressive strength of 55/45 to 70/30 cement/Rice Husk Ash (RHA) in hollow sandcrete block. The low cost and local availability potential of RHA, a pozzolanic material gasps for exploitation. The study applies the Scheffe's optimization approach to obtain a mathematical model of the form $f(x_{i1}, x_{i2}, x_{i3} x_{i4})$, where x are proportions of the concrete components, viz: cement, RHA, sand and water. Scheffe's i experimental design techniques are followed to mould various hollow block samples measuring 450mm x 225mm x 150mm and tested for 28 days strength. The task involved experimentation and design, applying the second order polynomial characterization process of the simplex lattice method. The model adequacy is checked using the control factors. Finally, a software is prepared to handle the design computation process to take the desired property of the mix, and generate the optimal mix ratios. Reversibly, any mix ratios can be desired and the attainable strength obtained.

Keywords: Sandcrete, Pseudo-component, Simplex-lattice, optimization, transformation matrix.

INTRODUCTION

Construction of structures is one of the most regular activities in life. It often involves sancrete block heavily both for load bearing or non-load bearing walls. The cost/stability of this operation has been a major issue in the world where cost is a major index.

Naturally, the rate of development in an area depends to a large extent on the usability of the raw materials in the process of environmental upgrading of that area as this translates to some fast and easy-cheap-to-find technology derived from innovative and testable ideas for technology growth.

In construction concrete is the main material. the cost of its production heavily impacts on construction and what can be done for the desired growth in the development of an area by way of construction of new roads, buildings, dams, water structures and the renovation of these. To produce the concrete several primary components such as cement, sand, gravel with or without admixtures are to be present in varying quantities and qualities. Unfortunately, the occurrence and availability of these components vary very randomly with location and hence the attendant problems of either excessive or scarce quantities of the different materials occurring in different areas. Hence, where the scarcity of one component is acute the cost of the concrete production increases relatively tremendously and such problems suggest the search for and application of the more readily available materials in full or part. This is the principal case of the study of the usability of laterite in place of sand or partial replacement of the sand under careful handling and modeling of predictive importance. This is of cost importance as long as the desired quality of the concrete product is not compromised.

The Concept of Optimization

Planning is the soul and success of every activity in human endeavour. The target of such planning is the maximization of the desired outcome of the execution of the plan. As agreed by Orie and Osadebe, this calls for minimum investments inputs and minimum outputs. The process involved minimization and maximization is referred to as optimization. In the science of optimization, the desired property or quantity to be optimized is referred to as the objective function. The raw materials or produce this objective function are referred to as variables.

Also there are constraining conditions called constraints. money, for instance, is a factor of production and is known to be limited in supply. Hence it is a constraint. Making structural concrete is not an all- corners affairs even though everybody can make concrete. Concrete is heterogeneous as it comprises sub-materials such as cement, fine aggregates, coarse aggregates, water, and sometime admixtures. David and Galliford (2000), reported that modern research in concrete seeks to provide greater understanding of its constituent materials and possibilities of improving its qualities.

Optimization of Concrete Mix

Genadij and Juris (1998) explained that the task of concrete mix optimization implies selecting the most suitable concrete aggregates from the data base. Several methods have been applied. Examples are by Mohan (2002), Simon (2003). Also, Bloom and Benture (1995) reports that optimization of mix designs require detailed knowledge of concrete properties.

Modeling

Modeling means setting up mathematical models/formulations of physical or other systems. Such models are constructed for the assessment of the objective function after using the hindsight of observed operating variables. Hence or otherwise, model could be constructed for a proper observation of response from the interaction of the factors through controlled fora proper observation of response from the interaction of the factors through controlled experimentation followed by schematic designed where such simplex lattice approach of the type of Scheffe (1958) optimization theory could be employed. Also entirely different physical systems may correspond to the same mathematical model so that they can be solved by the same methods. Erwin (2004) emphasizes that this is an impressive demonstration of the unifying power of mathematics.

Literature Review

Wilby (1963) observed that, to be a good structural material, the material should be homogeneous and isotropic. The Portland te cement, rice husk ash, laterite or concrete are none of these, nevertheless they are popular construction materials. Reynolds and Steedman (1981) maintain that, given the proportions of aggregates the compressive strength of concrete depends primarily upon age, cement content, and the cement-water ratio.

In his work on Optimum Design of Structure, Majid 91874) noted that, of all the desirable properties of hardened concrete such as the tensile, compressive, flexural, bond, shear strengths, etc., the compressive strength is the most convenient to measure and is used as the criterion for the overall quality of the hardened concrete.

Oluremi (1990) claimed that sandcrete blocks produced with RHA is widely acceptable to minimize the cost of construction works. Also a survey by the Raw Material Research and Development Council of Nigeria reveals that certain building materials deserve serious consideration as substitute for imported ones. Few of these include cement, lime stabilized blocks, adobe soil blocks, clay blocks, rice husk ash, lime and stonecrete blocks. According to Smith (1984). Rice husk, when burnt under controlled conditions, is highly pozzolanic and very suitable for use in lime-pozzolana mixes and for Portland cement replacement.

Defined by Jackson (1983), Simplex remains the structural representation (shape) of lines or planes joining assumed positions or points of the constituent materials (atom) of a mixture, and they are equidistant from each other. In their work on Experiment and Optimization in Chemistry and Chemical Engineering, Akhnazarov and Kafarov (1982) made it clear that the properties of q-component mixture, are dependent on the component ratio only the factor space is a regular (q-1) simplex. they explained that Simplex Lattice designs are saturated. That is, the proportions used for each factor have m + 1 equally spaced levels from 0 to 1 (x_i = 0, 1/m, 2/m, … 1), and all possible combinations are derived from such values of the component concentrations, that is, all possible mixtures, with these proportions are used.

Scheffe's Lattice Design Theory

This is a theory where a polynomial expression of any degrees, is used to characterize a simplex lattice mixture components. In the theory only a single phase mixture is covered. The theory lends path to a unifying equation model capable of taking varying component ratios to fix approximately equal mixture properties. The optimization, from economic view point, aims at selecting the optimal ratio from the component ratios list that can be automatically generated. Scheffe (1958) developed a model in which the response surfaces of the physical and chemical characteristics of a mixture can be approximated by a polynomial of the second and higher degrees.

Using the approach, Akhnazarova and Kafarov (1982) predicted the variations of reactivity and porosity of coke with the charges of four process groups of coal in a mixture. the approach could be adapted to predict the desired strength of concrete where the essential factors lies on the adequate proportioning of ingredients needed to make the concrete where with the compressive strength desired specified, possible combinations of needed ingredients to achieve the compressive strength can easily be predicted by the aid of computer, and if proportions are specified, the compressive strength can easily be fixed. In this paper, Scheffe's theory is adapted to formulate the response function for compressive strength of sandcrete with partial replacement of cement with rice husk ash.

The Simplex Lattice Method

Mathematically, a simplex lattice is a space of constituent variables of X_1, X_2, X_3, ...and X_i which obey these laws:

X_i =00 = X_1 = 1 .... (3.1)

?x_i = 1

A lattice is purely an abstract space to achieve the desired strength of concrete, one of the essential factors lied on the adequate proportioning of ingredients needed to make the concrete.

In designing experiment to attack mixture problems involving component property diagrams Akhnazarova and Kafarov (1982) assumed the property studied is assumed to be a continuous function of certain arguments and with a sufficient accuracy it was approximated with a polynomial. When investigating multi-components systems the use of experimental design methodologies substantially reduces the volume of an experimental effort. Further, this obviates the need for a special representation of complex surface, as the wanted properties can be derived from equations while the possibility to graphically interpret the result is retained. As a rule the response surfaces in multi-component systems are very intricate. To describe such surfaces adequately, high degree polynomials are required, and hence a great many experimental trials. A polynomial of degree n in q variable has C^{th}_{q+n} coefficients. If a mixture has a total of q the components and x_1 be the proportion of the i^{th} component in the mixture such that,

x_i= 0 (i=1 ,2, ….q),1 .... (3.2)

then the sum of the component proportion is a whole unity i.e.

$X_1 + X_2 + X_3 + X_4 = 1$ or ?X_i 1 = 0.....(3.3)

Thus the factor space is a regular (q-1) dimensional simplex. In (q-1) dimensional simplex if q = 2, we have 2 points of connectivity. This gives a straight line simplex lattice. If q=3, we have a triangular simplex lattice and for q = 4, it is a tetrahedron simplex lattice, etc. Taking a whole factor space in the design we have a (q,m) simplex lattice whose properties are defined as follows:

i. The factor space has uniformly distributed points,
ii. Simplex lattice designs are saturated (Akhnarova and Kafarov, 1982). That is, the proportions used for each factor have m + 1 equally spaced levels from 0 to 1 (x_i = 0, 1/m, i 2/m, … 1), and all possible combinations are derived from such values of the component concentrations, that is, all possible mixtures, with these proportions are used.

Hence, for the quadratic lattice (q,2), approximating the response surface with the second degree polynomials (m=2), the following levels of every factor must be used 0, ½ and 1; for the quadratic (m=4) polynomials, the levels are

0, 1/4, 2/4, 3/4 and 1, etc; Scheffe, (1958), showed that the number of points in a (q,m) lattice is given by

$C_{q+m-1} = q(q+1) \ldots (q+m-1)/m!$ .... (3.4)

The (4.2) Lattice Model

The properties studied in the assumed polynomial are real-valued functions on the simplex and are termed responses. The mixture properties were described using polynomials assuming a polynomial function of degree m in the q-variable X_1 , X_2 ……, X_q , subject to equation 3.1, and will be called a (q,m) polynomial having a general form: $Y = b_0 + \sum b_i X_i + \sum b_{ij} X_i X_j + \ldots + b_{ijk} +$ $\sum b_{i1i2 \ldots in} X_{i1} X_{i2} \ldots X_{in}$ .... (3.5)

$Y = b_0 + b_1 X_1 + b_2 X_2 + b_3 X_3 + b_4 X_4 + b_{12} X_1 X_2 + b_{13} X_1 X_3 + b_{14} X_1 X_4 + b_{24} X_2 X_4 + b_{23} X_2 X_3 + b_{34} X_3 X_4 \; b_{11} X_1^2 + b_{22} X_2^2 + b_{33} X_3^2 + b_{44} X_4^2$ .... (3.6)

where b is a constant coefficient.

The relationship obtainable from Eqn (3.6) is subjected to the normalization condition of Eqn. (3.3) for a sum of independent variables. For a ternary mixture, combining Eqns 3.3 and 3.6 with some mathematical re-arrangements the reduced second degree polynomial can be obtained as:

$\gamma = \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \beta_{12} X_1 X_2 + \beta_{12} X_1 X_3 + \beta_{13} X_1 X_3 + \beta_{14} X_1 X_4 + \beta_{23} X_2 X_{23} + \beta_{24} X_2 X_4 + \beta_{34} X_3 X_4$ .... (3.7)

where $\beta_1 = \beta_0 X_2 X_{23}$

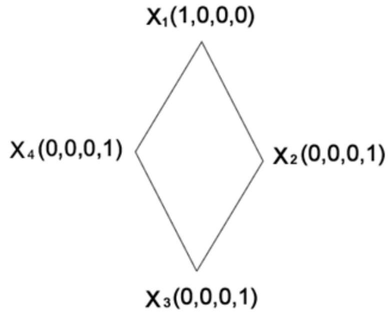
and $\beta_{ij} X_{ij} X_{ij}$

That is;

$Y = \sum \beta_i X_i + \sum \beta_{ii} X_i$ .... (3.8)

Construction of Experimental/Design Matrix

From the coordinates of points in the simplex lattice, we can obtain the design matrix. We recall that the principal coordinates of the lattice, only a component is 1 (refer to fig 3.1), others are zero.



Tetrahedral Simplex Lattice

Table 3.1 Pseudo Proportions and Property Functions

N	X_4	X_2	X_3	X_1	RESPONSE
1	1	0	0	0	Y_1
2	0	1	0	0	Y_2
3	0	0	1	0	Y_3
4	0	0	0	1	Y_4
5	½	½	0	0	Y_{12}
6	½	0	½	0	Y_{13}
7	½	0	½	0	Y_{14}
8	0	½	½	0	Y_{23}
9	0	½	0	½	Y_{24}
10	0	0	½	½	Y_{34}

we get that $Y_{12} = \hat{a}_{1+}$

And doing so in succession for the other three points if the hexahedron, we obtain

$Y_{12} = \beta_2 + X_3 = 0$, and $X_4 = 0$ .... (3.9)

the substitution of the coordinates of the fifth experimental point yields

$Y_{12} = ½ X_1 + ½ X_2 + ½ X_1 ½ X_2$

$= ½ \beta_1 + ½ \beta_2 + ½ \beta_{12}$

Thus

$_2 = 4Y_{12} - 2Y_1 - 2Y_1 - 2Y_2$ .... (3.10)

and similarly,

$B_{13} = 4Y_{13} - 2Y_1 - 2Y_2$

$B_{23} = 4Y_{23} - 2Y_2 - 2Y_3$

Or generalizing,

$\hat{a}, = Y_i$, and $\beta_{ij} = 4Y_{ij} - 2Y_i - 2Y_j$........ (3.10) which are the coefficients of the reduced second degree polynomial for a q-component mixture, since the four points defining the coefficients $\hat{a}_{ij}$ lie on the edge. The subscripts of the mixture property symbols indicate the relative content of each component X the property of the mixture is denoted by Y. Mixture 5 includes X_1 and X_2 , and the property is designated Y_{12}.

Actual and Pseudo Components

The requirements of the simplex that

$\sum X_i = 1$.................. (3.11)

makes it impossible to use the normal mix ratios such as 1:3, 1:5, etc, at a given water/cement ratio. Hence a transformation of the actual components (ingredient proportions) to meet the above criterion is unavoidable. Such transformed ratios say $X^{(i)}_1$ $X^{(i)}_2$ and $X^{(i)}_3$ for the i^{th} experimental points are called pseudo components. Since X_1 , X_2 and X_3 are subject to ZXi = 1, the transformation of cement: RHA: sand: water at say 0.45 water/cement ratio cannot easily be computed because X_1 , X_2 and X_3 are in pseudo expressions $X^{(i)}_1$ $X^{(i)}_2$ and $X^{(i)}_3$. For the i^{th} experimental point, the transformation computations are to be done by some multiplicative operations between the pseudo and the initially arbitrarily assumed actual variables.

The arbitrary vertices chosen on the triangle are A_1(0.6:0.4:8:0.45), A_2(0.66:0.34:8.5:0.4), A_3(0.7:0.3:8.1:0.48) and A_4(0.55:0.45:7.8:0.50), based on experience and earlier research reports.

Transformation Matrix

If Z denotes the actual matrix of the i^{th} experimental points, observing from Table 3.2 (points 1 to 3),

BZ = X =1 .... (3.12)

where B is the transformed matrix.

Therefore, $B = I.Z^{-1}$

Or $B=Z^{-1}$ .... (3.13)

The inverse transformation from pseudo component to actual component is expressed as

AX = Z .... (3.14)

where A = inverse matrix

$= Z X^{-1}$.

from Eqn. 3.12, X = BZ, therefore,

A = Z, (BZ)-1

$A = Z, Z\text{-}1\ B^{-1}$

$A = IB^{-1}$

$A = B^{-1}$

Table 3.2 Values for Experiment

Eqn 3.14 is used to determine the actual components from points 5 to 10, and the control values from points 11 to 13 (Table 3.2)

N	X_1	X_2	X_3	X_4	RESPONSE	Z_1	Z_2	Z_3	Z_4
1	1	0	0	0	Y_1	1.00	0.49	16.66	0.49
2	0	1	0	0	Y_2	1.00	0.45	14.95	0.33
3	0	0	1	0	Y_3	1.00	0.50	14.89	0.37
4	0	0	0	1	Y_4	1.00	5.00	16.83	0.45
5	½	½	0	0	Y_{12}	1.00	0.47	15.78	0.34
6	½	0	½	0	Y_{13}	1.00	0.50	15.75	0.31
7	½	0	½	0	Y_{14}	1.00	2.75	16.72	0.35
8	0	½	½	0	Y_{23}	1.00	0.48	14.92	0.33
9	0	½	0	½	Y_{24}	1.00	2.73	15.89	0.37
10	0	0	½	½	Y_{34}	1.00	2.75	15.86	0.33
					control points				
11	0.25	0.25	0.25		Y_{1234}	1.00	1.61	15.82	0.34
12	0.5	0.25	0.25		Y_{1123}	1.00	0.48	15.76	0.32
13	0.25	0.5	0		Y_{1224}	1.00	1.60	15.83	0.35

Use of Values in Experiment

During the laboratory experiment, the actual components were used to measure out the appropriate proportions of the ingredients: cement, rice husk ash, sand and water, for mixing the lateritic concrete materials for casting the samples. The values obtained are presented in Tables in section 5.

Adequacy of Tests

This is carried out by testing the fit of a second degree polynomial. After the coefficients of the regression equation has been derived, the statistical analysis is considered necessary, that is, the equation should be tested for goodness of fit, and the equation and surface values bound into the confidence intervals. In experimentation following simplex-lattice designs there are no degrees of freedom to test the equation for adequacy, so, the experiments are run at additional so-called test points.

The number of control points and their coordinates are conditioned by the problem formulation and experiment nature. Besides, the control points are sought so as to improve the model in case of inadequacy. The accuracy of response prediction is dissimilar at different points of the simplex. The variance of the predicted response, S^2_y, is obtained from the error accumulation law. To illustrate this by the second degree polynomial for a ternary mixture, the following points are assumed: X can be observed without errors (Akhanarova and Kafarov, 1982).

The replication variance, S^2_y, is similar at all design points, and response values are the average of n and n replicate observations at appropriate i ij points of the simplex

Then the variance S_{yi} and S_{yij} will be

$(S_y^2)_I = S_y^2 / n_i$ .... (3.16)

$(S_y^2)_{ij} = S_y^2 / n_{ij}$ .... (3.17)

In the reduced polynomial,

$Y = \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \beta_{12} X_1 X_2 + \beta_{13} X_1 X_3 + \beta_{14} X_1 X_4 + \beta_{23} X_2 X_{23} + \beta_{24} X_2 X_4 + \beta_{34} X_3 X_4$ ....(3.18)

if we replace coefficients by their expressions in terms of responses in

$\beta_i = Y_i$ and $\beta_{ij} = 4Y_{ij} - 2Y_i - 2Y_j$ .... (3.19)

using the condition $X_1 + X_2 + X_3 + X_4 = 1$, and rearranging terms, we obtain that Y =

$X_1 (2X_1 1) Y_1 + X_2 (2X_2 1) Y_2 + X_3 (2X_3 1) Y_3 + X_4 (2X_4 1) Y_4 + 4Y_{12} X_1 X_2 + 4Y_{13} X_1 X_3 + 4Y_{14} X_1 X_4 + 4Y_{23} X_2 X_3 + 4Y_{24} X_2 X_4 + 4Y_{34} X_3 X_4$ .... (3.20)

Introducing the designation

$a_i = X_i(2X_i - 1)$ and $a_{ij} = 4X_iX_j$ .... (3.21)

and using Eqns (3.20) and (3.21) give the expression for the variance S_y^2.

$S_y^2 = S_y^2 \ (\sum a_{ii} / n_i + \quad a_{jj} / n_{ij}) \ \sum$..... (3.22)

If the number of replicate observations at all the points of the design are equal, i.e. $n_i = n_{ij} = n$, then all the relations for S_{y2} will take the form

$S_{y2} = S_{y2}$ i/n .... (3.23)

where, for the second degree polynomial,

$\xi = \sum_{1 \leq i \leq q} a_i^2 + \quad a_{ij}^2$ ....$\sum_{1 \leq i \leq iq}$.... (3.24)

As in Eqn (3.23), ξ is only dependent on the mixture composition. Given the replication Variance and the number of parallel observations n, the error for the predicted values of the response is readily calculated at any point of the composition-property diagram using an appropriate value of ξ taken from the curve.

Adequacy is tested at each control point, for which purpose the statistic is built:

$t = \triangle_y / (SS_{y2} + SS_{y2}) = \ _y n^{1/2} / (S_y (1 + \xi)^{1/2}$ ..$\triangle$. (3.25)

where $\triangle_y = Y_{exp} - Y_{theory}$ .... (3.36)

and n = number of parallel observations at every point.

The t-statistic has the student distribution, and it is compared with the tabulated value of $t_{\alpha/L}$ (V) at a level of significance α, where L = the number of control points, and V = the number for the degrees of freedom for the replication variance.

The null hypothesis is that the equation is adequate is accepted if $t_{cal} < t_{Table}$ for all the control points.

The confidence interval for the response value is

$$Y - \Delta = Y = Y + ? \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3.27)$$

where $\Delta = t_{\alpha/L,k}\, S_y$ .... (3.28)

and k is the number of polynomial coefficients determined.

Using Eqn (3.29) in Eqn (3.28)

$$D = t_{\alpha/L,k}\, S_y\, (\xi/n) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3.29)$$

MATERIAL AND METHODS

Materials

Rice husk materials were collected from the waste rice husk dump site of Olo rice mill at Ezeagu Local Government area of Enugu State.

The water for use is pure drinking water which is free from any contamination i.e. nil Chloride content, pH =6.9, and Dissolved Solids < 2000ppm. Ordinary Portland cement is the hydraulic binder used in this project and source from the Dangote Cement Factory, and assumed to comply with the Standard Institute of Nigeria (NIS) 1974, and kept in an air-tight bag. The sand was got from the Iyioku river basin and conformed to a maximum size of 1mm.

Equipment

The equipment used in the study include block moulds, tampers, weighing scale and crushing machine.

Preparation of Samples

The sourced materials for the experiment were transferred to the laboratory. The pseudo components of the mixes were designed following the background theory from where the actual variables were developed. The component materials were mixed at ambient temperature according to the specified proportions of the actual components generated in Table 3.2. In all, two solid blocks of 450mm x225 x150mm for each of ten experimental points and three control points were cast for the compressive strength test, cured for 28 days after setting and hardening.

Strength Test

After 28 day of curing, the and blocks were crushed to determine the sandcrete block strength, using the testing machine to the requirements of BS 1881:Part 115 of 1986.

RESULTS AND ANALYSIS

Determination of Replication Error And Variance of Response

To raise the experimental design equation models by the lattice theory approach, two replicate experimental observations were conducted for each of the ten design points. Hence we have below, the table of the results (Tables 5.1) which contain the results of two repetitions each of the 10 design points plus three Control Points of the (4,2) simplex lattice, and show the mean and variance values per test of the observed response, using the following mean and variance equations below:

$$I\ \ddot{Y} = \sum(Y_r)/r \quad \ldots\ldots\ \ldots\ldots\ldots\ldots\ldots \quad 5.1$$

where Y is the mean of the response values and $_n$r =1,2. Table

$$S_y^2 = \sum[(Y_i - \ddot{Y}_i)^2]/(n-1) \quad \ldots\ldots\ldots\ldots\ldots \quad 5.2$$

where n = 13

Table 5.1 Result of the Replication Variance of the Compressive Strength Response for 450mm x225 x150mm Block

Experiment no(n)	Repetition	Response f_e(N/mm^2)	Response Symbol	?Y_r	$\ddot{Y}_r$	?(Y_r - $\ddot{Y}_r$)	S$\ddot{Y}_i\ddot{Y}^2$
1	1A	0.91					
	1B	1.01	Y_1	1.92	0.96	0.01	0.00
2	2A	0.68					
	2B	0.77	Y_2	1.45	0.73	0.00	0.00
3	3A	1.11					
	3B	0.90	Y_3	2.01	1.01	0.02	0.00
4	4A	0.87					
	4B	0.59	Y_4	1.46	0.73	0.04	0.00
5	5A	0.70					
	5B	1.13	Y_{12}	1.83	0.92	0.09	0.01
6	6A	1.07					
	6B	0.81	Y_{13}	1.88	0.94	0.03	0.00
7	7A	0.94					
	7B	1.09	Y_{14}	2.03	1.02	0.01	0.00
8	8A	0.69					
	8B	0.94	Y_{23}	1.63	0.82	0.03	0.00
9	9A	0.85					
	9B	0.87	Y_{24}	1.72	0.86	0.00	0.00
10	10A	1.00					
	10B	0.95	Y_{34}	1.95	0.98	0.01	0.00
			control points				
11	11A	1.03					
	11B	0.71	C_{1234}	1.74	0.87	0.05	0.00
12	12A	0.98					
	12B	0.87	C_{1123}	1.85	0.93	0.01	0.00
13	13A						
	13B		C_{1224}	1.83	0.92	0.04	0.00

Replication Variance

$S_y^2 = \sum S_i^2 = 0.33$

That's

Replication error $S_y = (0.33/13\text{-}1))^{½} = 0.17$

5.2 Determination of Regression Equation for the Compressive Strength

From Eqns 3.15 and Table 5.1 the coefficients of the reduced second degree polynomial is determined as follows:

$\beta_1 = 0.96$

$\beta_2 = 0.73$

$\beta_3 = 1.01$

Total: 0.33 0.01

$\beta_{12} = 4(0.92) - 2(0.96) - 2(0.73) = 0.29$ 12

$\beta_{13} = 4(0.94) - 2(0.96) - 2(1.01) = 0.17$ 13

$\beta_{14} = 4(1.02) - 2(1.96) - 2(0.73) = 0.70$ 14

$\beta_{23} = 4(0.82) - 2(0.73) - 2(1.01) = 0.20$ 23

$\beta_{24} = 4(0.86) - 2(0.73) - 2(1.73) = 0.53$ 24

$\beta_{34} = 4(0.98) - 2(1.01) - 2(1.92) = 0.43$ 34

Thus, substituting in Eqn (3.20),
$Y = 0.96X_1 + 0.73X_2 + 1.01X_3 + 0.73X_4 + 0.29X_1 X_2 - 0.17X_1 X_3 + 0.70X_1 X_4 - 0.20X_2 X_3 + 0.53X_2 X_4 + 0.43X_3 X_3$ .... (5.3)

Eqn 5.3 is the mathematical model of the compressive strength of hollow sandcrete block based on 28-day strength.

Test of Adequacy of the Compressive strength

Eqn 5.3, the equation model, will be tested for adequacy against the controlled experimental results. We recall our statistical hypothesis as follows:

1. Null Hypothesis (H): There is no significant difference between the experimental values and the theoretical expected results of the compressive strength.
2. Alternative Hypothesis (H): There is a significant difference between the experimental values and the theoretical expected results of the compressive strength.

If we substitute for X_i in Eqn 5.4 from Table 3.3, t he theoretical predictions of the response (Y) can be obtained. These values can be compared with the experimental results (Table 5.1). For the t- test (Table 5.2), a, ξ, t and $?_y$ are evaluated using Eqns 3.31, 3.32, 3.35, 3.27a and 3.30 respectively.

N	CN	I	J	a_i	a_{ij}	a_i^2	a_{ij}^2	ξ	Ÿ	Y	$?_y$	t
1	C_1	1	2	-0.125	0.250	0.016	0.063					
		1	3	-0.125	0.250	0.016	0.063					
		1	4	-0.125	0.250	0.016	0.063					
		2	3	-0.125	0.250	0.016	0.063	0.469	0.87	0.95	-0.08	-0.57
		2	4	-0.125	0.250	0.016	0.063					
		3	4	-0.125	0.250	0.016	0.063					
				0.094	0.375							
2	C_2	1	2	0.000	0.500	0.000	0.250					
		1	3	0.000	0.500	0.000	0.250	0.563	0.93	0.92	0.02	0.10
		1	4	0.000	0.000	0.000	0.000					
		2	3	0.000	0.250	0.000	0.063					
		2	4	0.000	0.000	0.000	0.000					
		3	4	0.000	0.000	0.000	0.000					
					0.000	0.563						
3	C_3	1	2	-0.125	0.500	0.016	0.250					
		1	3	-0.125	0.000	0.016	0.000					
		1	4	-0.125	0.250	0.016	0.063					
		2	3	-0.125	0.000	0.016	0.000	0.656	0.92	0.93	-0.01	-0.07
		2	4	-0.125	0.500	0.016	0.250					
		3	4	-0.125	0.000	0.016	0.000					
					0.094	0.563						

Significance level $\alpha = 0.05$,

i.e. $t_{\alpha/L}$ (V) $= t_{0.05/3}$ (13), where L=number of control points.

From the Student's t-table, the tabulated value of $t_{\alpha/L}$ (V) $= t_{0.05/3}$ (13) is found to be 2.450 which is greater than the calculated t-values in Table 5.2. Hence we can accept the Null Hypothesis.

From Eqn 3.35, with k=10 and $t_{\alpha/L} = t_{v\ 0.05/k}$ (13) = 3.01,

D = 0.61 for C_{1234} , 0.63 for $C_{1124\ =0.26,}$, and 0.65 for $C_{1224,}$

which satisfies the confidence interval equation of Eqn. 3.33 when viewed against the response values in Table 5.2.

Computer Program

The computer program is developed for the model). In the program any desired Compressive Strength can be specified as an input and the computer processes and prints out possible combinations of mixes that match the property, to the following tolerance:

Compressive Strength - 0.00005 N/mm^2 ,

Interestingly, should there be no matching combination, the computer informs the user of this. It also checks the maximum value obtainable with the model.

```
        'QBASIC BASIC PROGRAM THAT OPTIMIZES THE PROPORTIONS OF SANDCRETE MIXES
        USING THE SCHEFFE'S MODEL FOR CONCRETE COMPRESSIVE STRENGTH
        CLS
        CLS = "(ONUAMAH.HP) RESULT OUTPUT": C25 = " A COMPUTER PROGRAM"
C3$ = "ON THE OPTIMIZATION OF A 4-COMPONENT SANDCRETE MIX"
PRINT C2$ + C1$ + C3$
PRINT
        VARIABLES USED ARE
        'X1, X2, X3, X4, Z1, Z2, Z3, Z4, Z$, YT, YTMAX, DS

INPUT" ARE MIX RATIOS KNOWN AND THE ATTAINABLE STRENGTH NEEDED?
CHOOSE Y = YES OR N = NO":
OPTIONS
  IF OPTIONS = "Y" THEN
  INPUT "X1 = "; X1
  INPUT "X2 = "; X2
  INPUT "X3 = "; X3
  INPUT "X4 = "; X4
  X = X1 + X2 +X3 + X4
  X1 = X1 /X: X2 = X2 /X: X3 = X3 /X: X4 = X4 / X
  GOTO 25
ELSE
  GOTO 20
END IF

  'INITIALISE 1 AND YTMAX

20 1 =0: YTMAX = 0

  FOR MXI = 0 TO 1 STEP .01
  FOR MX2 = 0 TO 1 - MXI STEP .01
  FOR MX3 = 0 TO 1 MXI - MX2 STEP .01
  FOR MXI = 0 TO 1 STEP .01
   MX4 = 1 - MX1 - MX 2 - MX3
    YTM = .96 * MX1 + .73 * MX2 + 1.01
*MX3 + .73 * MX4 + .29 * MX1 * MX2 - .17
*MX1 * MX3 + .70 * MX1 * MX4 - .2 *MX2
*MX3 + .53 * MX2 * MX4 + .43 * MX3*MX4
IF YTM >+ YTMAX THEN YTMAX = YTM
  NEXT MX3
  NEXT MX2
  NEXT MX1
  INPUT "ENTER DESIRED STRENGTH, DS = ";DS

'PRINT OUTPUT HEADING PRINT
25   PRINT TAB (1); "NO"; TAB(10); "X1";
TAB (18); "X2"; TAB(26); "X3";
TAB(34); "X4"; TAB(40); "YTHEORY";
TAB(50); "Z1";
TAB(58); "Z2"; TAB(64); "Z3"; "Z4";
  IF OPTIONS = "Y" THEN 30
  PRINT
COMPUTE THEORETICAL STRENGTH, YT
  FOR X1 = 0 TO 1 STEP .01
   FOR X2 = 0 TO 1 - X1 STEP .01
    FOR X3 = 0 TO 1 - X1 - X2 STEP .01
X4 = 1 - X1 - X2 - X3
30        YT = .96 * X1 + .73 * X2 + 1.01 * X3
+.73 * X4 + .29 * X1 * X2 - .17 * X1 * X3 + .70
* X1 + * X4 -.2 * X2 * X3 + .53 * X2 * X4 +
.43 * X3 * X4
```

```
IF OPTIONS = "Y" THEN 40
IF ABS(YT - DS) <= .00005 THEN
'PRINT MIX PROPORTION
RESULTS
Z1 = X1 + X2 + X3 + X4: Z2 = 6.01 * X1 +
6.07 * X2 + 2.54 * X3 + 6.75 * X4; Z3 2.9 *
X1 + 2.93 * X2 + 2.54 * X3 + 3.25 * X4: Z4 =
.3 * X1 + .45 * X2 + .45 * X3 + .5 * X4
40    1 = 1 + 1
     PRINT TAB(1); 1; USING "##.###";
TAB(7); X1
TAB(15); X2; TAB(23); X3;
TAB(32); X4; TAB(40); YT;
TAB(48); Z1; TAB(56); Z2;
TAB(62); Z3; TAB(70); Z4
   PRINT
   PRINT
IF OPTION$ = "Y" THEN 540
   IF (X1 = 1) THEN 550
   ELSE
   IF (X1 < 1) THEN GOTO 150
   END IF

150     NEXT X3
   NEXT X2
   NEXT X1
  IF I > 0 THEN 550
   PRINT
   PRINT "SORRY, THE DESIRED
STRENGTH IS OUT OF RANGE OF
MODEL"
   GOTO 600

550 PRINT TAB(5); "THE ATTAINABLE
STRENGTH IS"; YT; ;N/nm2"
  GOTO 600
550 PRINT TAB(5); "THE MAXIMUM
VALUE PREDICTABLE BY THE MODEL
IS ";
YTMAX; "N / Sq mm "
600 END
```

```
A COMPUTER PROGRAM
(ONUAMAH.COM) RESULT OUTPUT ON
THE OPTIMIZATION OF A 4-
COMPONENT SANDCRETE MIX ARE
MIX RATIOS KNOWN AND THE
ATTAINABLE STRENGTH NEEDED?,
CHOOSE Y = YES OR N = NO?
N ENTER DESIRED STRENGTH, DS = ? 1.03
No    X1       X2       X3       X4
YTHEORY  Z1       Z2       Z3       Z4
1     0.590    0.050    0.020    0.340 1.030
      1.000    6.250    3.013    0.379
2     0.720    0.040    0.000    0.240 1.030
      1.000    6.190    9.985    0.354

THE MAXIMUM VALUE PREDICTABLE BY
THE MODEL IS 1.03888 N / Sq mm
Press any key to continue
```

Choosing a Combination

It can be observed that the strength of 1.03 N/sq mm yielded 4 combinations. To accept any particular proportions depends on the factors such as work ability, cost and honeycombing of the resulting concrete.

CONCLUSION /RECOMMENDATION

Scheffe's (1958) simplex design was applied successfully to prove that the modulus of lateritic concrete is a function of the proportion of the ingredients (cement, cement, RHA, water), but not the quantities of the materials.

The maximum compressive strength obtainable with the compressive strength model is 1.03888 N/sq mm. See the computer run outs which show all the possible lateritic concrete mix options for the compressive strength of 1.03 2N/mm_2 , and the choice of any of the mixes is the user's.

One can also draw the conclusion that the maximum values achievable, within the limits of experimental errors, is quite below that obtainable using only cement. This is due to the low binding strength of RHA.

It can be observed that the task of selecting a particular mix proportion out of many options is not easy, if work ability and other demands of the resulting lateritic concrete have to be satisfied. This is an important area for further research work.

iii) More research work need to be done in order to match the computer recommended mixes with the workability of the resulting concrete, the accuracy of the model can be improved by taking higher order polynomials of the simplex.

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